1\. (a) Given the matrices $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$,

determine the matrix $C = 3A^2 - 4B$. (5 marks)

(b) Solve the equation

$$\begin{vmatrix} x-5 & 2 \\ 2 & x-2 \end{vmatrix} = 0$$

(5 marks)

(c) Three currents $I_1$, $I_2$ and $I_3$ in amperes flowing in a d.c. network, satisfy the simultaneous equations:

$$\left.\begin{aligned} I_1 - 2I_2 + I_3 &= -3 \\ -2I_1 + I_2 + I_3 &= 0 \\ I_1 + 3I_2 - 2I_3 &= 9 \end{aligned}\right\}$$

Use the inverse matrix method to determine the values of the currents. (10 marks)

2\. (a) Find the seventh term in the binomial expansion of $(4+3x)^{12}$, and determine its value when $x = \frac{1}{3}$. (6 marks)

(b) Determine the first four terms in the binomial expansion of

$$\sqrt{\left(\frac{1-x}{1+x}\right)}.$$

(7 marks)

(c) (i) Use the binomial theorem to expand $(1-8x)^{\frac{1}{2}}$ as far as the term in $x^3$.

(ii) Hence, by setting $x = \frac{1}{100}$, determine the value of $\sqrt{23}$, correct to four decimal places. (7 marks)

3\. (a) Find $\frac{dy}{dx}$ from first principles, given $y = \frac{1}{x-3}$. (5 marks)

(b) Use implicit differentiation to determine the values of:

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

at the point $(1,1)$ for the curve $x^2 + y^2 + 3x - 4y = 1$. (8 marks)

(c) Determine the stationary points of the curve $f(x) = x^3 + 15x^2 + 27x - 6$, and state their nature. (7 marks)

4. (a) Prove the identities:

(i) $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=2\sec\theta$

(ii) $\cos 3x = 4\cos^3 x - 3\cos x$. (8 marks)

(b) Eliminate $\theta$ from the equations:

$x=\sec\theta,\ y=\cos 2\theta$. (3 marks)

(c) Solve the equation:

$\cos 2\theta + \sin\theta = 0$, for values of $\theta$ between $0^\circ$ and $360^\circ$ inclusive. (9 marks)

$\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}$

$\dfrac{(1+\sin\theta)(1+\sin\theta)+\cos\theta(\cos\theta)}{\cos\theta(1+\sin\theta)}$

$\dfrac{1+\sin\theta+\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$\dfrac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

but $\sin^2\theta + \cos^2\theta$

$\dfrac{1+2\sin\theta+1}{\cos\theta(1+\sin\theta)}$

$\dfrac{2\sin\theta+2}{\cos\theta(1+\sin\theta)}$

$\dfrac{2(\sin\theta+1)}{\cos\theta(1+\sin\theta)}$

$=2\sec\theta$

5. (a) Given that $3\sinh x + 5\cosh x = Me^x + Ne^{-x}$, determine the values of M and N. (4 marks)

(b) Prove the identities:

(i) $\cosh^2 x - \sinh^2 x = 1$

(ii) $\sinh 3x = 3\sinh x + 4\sinh^3 x$. (10 marks)

(c) Solve the equation:

$3\cosh 2x - 8\sinh x + 2 = 0$. (6 marks)

$\cos 3x = \cos(2x + x)$

$\cos 2x\cos x - \sin 2x \sin x$

$(2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x$

$2\cos^3 x - \cos x - 2\sin^2 x\cos x$

$2\cos^3 x - \cos x - (2 - 2\cos^2 x)\cos x$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$4\cos^3 x - 3\cos x$

6. (a) Evaluate the integrals:

(i) $\displaystyle\int_0^1 \frac{x^{-2}+2x^{-1}+1}{x^{-2}}\,dx$

(ii) $\displaystyle\int_0^{\pi} (4\sin x - 3\cos x + 2x)\,dx$ (8 marks)

(b) Use integration to determine the area of the region in the first quadrant bounded by the curve $y = x - x^2$, the $x-$ axis and the straight lines $x = 0$ and $x = 1$. (4 marks)

$3\sinh x + 5\cosh x - Me^x + Ne^{-x}$

$3\left(\frac{e^x - e^{-x}}{2}\right) + 5\left(\frac{e^x + e^{-x}}{2}\right) = Me^x + Ne^{-x}$

$\frac{3}{2}e^x - \frac{3}{2}e^{-x} + \frac{5}{2}e^x + \frac{5}{2}e^{-x} = $

$\frac{3}{2}e^x + \frac{5}{2}e^x = Me^x$

$\frac{8}{2}e^x = Me^x$

$M = 4$

$\frac{-3}{2}e^{-x} + \frac{5}{2}e^{-x} = Ne^{-x}$

$\frac{2}{2}e^{-x} = Ne^{-x}$

$N = 1$

(c) Three forces, $F_1$, $F_2$ and $F_3$ in newtons, necessary for the equilibrium of a system, satisfy the simultaneous equations:

$F_1 + 2F_2 - F_3 = 3$

$F_1 + F_2 + 2F_3 = 11$

$2F_1 - F_2 + F_3 = 6$

Use elimination method to solve the equations. (8 marks)

(3c) $x^3 + 15x^2 + 27x - c$

$\frac{dy}{dx} = 3x^2 + 30x + 27 = 0$

$3x^2 + 30x + 27 = 0$

$(3x + 3)(x + 9) = 0$

$3x + 3 = 0$

$3x = -3$

$x = -1$

Stationary Points $(-1, -9)$

1521/203 1601/203
1522/203 1602/203
June/July 2018

Turn over

7. (a) Given $u(x,y) = \sin x + \cos y$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + u = 0.$$

(5 marks)

(b) Use partial differentiation to determine the percentage change in the volume of a cylinder if its radius increases by 2% and its height decreases by 1%. (6 marks)

(c) Locate the stationary points of the function $z = x^2 - y^2 + 4xy - 2x + 6y$, and determine their nature. (9 marks)

8. (a) Given the vectors $\underset{\sim}{A} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{B} = -3\underset{\sim}{i} + 2\underset{\sim}{j} + 4\underset{\sim}{k}$, determine:

(i) a unit vector perpendicular to $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(ii) the angel between $\underset{\sim}{A}$ and $\underset{\sim}{B}$ (12 marks)

(b) Temperature distribution in a workshop is given by the scalar function $T(x,y,z) = x^2y + z^2$. Determine at the point (1,1,2):

(i) $|\nabla T|$;

(ii) $\nabla . \nabla T$.

(8 marks)

$A \cdot B = |A|\, B \cos\theta$

$A \cdot B = 2i - 3j + k$
$\quad -3i + 2j + 4k$
$-6 + 6 + 4$
$= -8$

$|A| = (2^2 - 3^2 + 1^2)^{\frac{1}{2}} \sqrt{4+9+1}$

$|B| = (-3^2 + 2^2 + 4^2)^{\frac{1}{2}} \sqrt{9+4+}$

$|A| = \sqrt{14}$

$|B| = \sqrt{29}$

$A \cdot B = |A||B| \cos\theta$

$+8 = \sqrt{14} \cdot \sqrt{29} \cos\theta$

$\sqrt{14}\sqrt{29} \quad \sqrt{14} \cdot \sqrt{29}$

$\cos^{-1}\theta = \frac{-8}{\sqrt{14}\sqrt{29}}$ 0.39

$A = \begin{pmatrix} 1 & -2 & 3 \\ 1 & -1 & 1 \\ 0 & 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 2 & 1 & -1 \end{pmatrix} \quad 4 \begin{pmatrix} 4 & 4 & 0 \\ -4 & 8 & 4 \\ 8 & 4 & -4 \end{pmatrix}$

def the matric $C = 3A^2 - 4B$

$3 \begin{pmatrix} 1 & -4 & 9 \\ 1 & -1 & 1 \\ 0 & 1 & 4 \end{pmatrix}$

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$\begin{pmatrix} 3 & -12 & 27 \\ 3 & -3 & 3 \\ 0 & 3 & 12 \end{pmatrix} + \begin{pmatrix} 4 & 4 & 0 \\ -4 & 8 & 4 \\ 8 & 4 & -4 \end{pmatrix} \begin{pmatrix} -1 & -16 & 27 \\ 7 & -11 & -1 \\ -8 & -1 & 16 \end{pmatrix}$

(ii) $\Delta T$

$i \quad j \quad k$

$\frac{dt}{dx} \quad \frac{dt}{dy} \quad \frac{dt}{dz}$

$2xy \quad x^2+z^2 \quad 2z$

$(2z - (x^2+z^2)) - j(2z - 2xy) + k(x^2+z^2 + 2xy)$

$i(2\times2 - 1 + 1) - j(2\times2 - 2) + k(1 + 4 - 2\times1\times1)$

$i(4) j(1) k(3)$

$4i + k3 \quad 4i + 3k$

$i\left(\frac{dt}{dy}(2z) - \frac{dt}{dz}(x^2+z^2)\right)$

$j\left(\frac{dt}{dx}(2z) - \frac{dt}{dz}(2xy)\right)$

(b) $(\Delta T)$

$T = x^2y + z^2$

(i) $i \quad j \quad k$

$i\left(\frac{dt}{dx}\right) = (x^2 + z^2)$

$\frac{dt}{dz} = 2z$

$i(2xy) - j(x^2+z^2) + k(2z$

1. (a) Prove the identities;

(i) $\dfrac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$

(ii) $\cos x + \cos 2x + \cos 3x = \cos 2x\,(2\cos x + 1)$. (8 marks)

(b) Eliminate $\theta$ from the equations:
$x = \tan\theta$ , $y = 2\sec^2\theta - 1$ (3 marks)

(c) Solve the equation:

$3\sin^2\theta + 4\cos\theta = 4$, for values of $\theta$ between $0^\circ$ and $360^\circ$ inclusive. (9 marks)

2. (a) Find $\dfrac{dy}{dx}$ from first principles, given $y = \dfrac{1}{2-x}$. (5 marks)

(b) The radius of a sphere is increased from 10 cm to 10.2 cm. Use differentiation to determine the approximate increase in the surface area, giving the answer as a percentage. (5 marks)

(c) Locate the stationary points of the function $f(x) = x^3 - 12x^2 + 36x - 6$, and determine its maximum and minimum values. (10 marks)

3. (a) Write down the middle term in the binomial expansion of $(2x+3)^8$ , and determine its value when $x = \dfrac{1}{3}$. (6 marks)

(b) Determine the first four terms in the binomial expansion of $(1-2x)^{\frac{1}{3}}$. (3 marks)

(c) The radius of a cylinder is reduced by 3% and its height is increased by 2%. Use the binomial theorem to determine the approximate percentage change in its:

(i) Volume;
(ii) Curved surface area. (11 marks)

4. (a) Give the matrices $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 1 \end{bmatrix}$, determine $(AB)^{-1}$. (9 marks)

(b) Three forces $F_1$, $F_2$ and $F_3$ in Newtons, acting an electromechanical system satisfy the simultaneous equations:

$-4 - 4 + 4 =$

$F_1 - 2F_2 + 2F_3 = 1$ $= 4$

$-2F_1 + F_2 + F_3 = 2$

$-F_1 + F_2 - 3F_3 = -5.$

$-4 + 2 - 6 =$

Use Cramer's rule to solve the equations. (11 marks)

$1 - 4 + 4 =$

$4 + 16 + 25$

$4^2 - 3^2 + 3^2$

$16 - 9$

$3F_2 = 14F_3$

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1. (a) Prove the identities;

(i) $\dfrac{1+\cos\theta}{1-\cos\theta}=(\operatorname{cosec}\theta+\cot\theta)^2$

(ii) $\cos x+\cos 2x+\cos 3x=\cos 2x\,(2\cos x+1)$. (8 marks)

(b) Eliminate $\theta$ from the equations:
$x=\tan\theta$ , $y=2\sec^2\theta-1$ (3 marks)

(c) Solve the equation:

$3\sin^2\theta+4\cos\theta=4$, for values of $\theta$ between $0^\circ$ and $360^\circ$ inclusive. (9 marks)

2. (a) Find $\dfrac{dy}{dx}$ from first principles, given $y=\dfrac{1}{2-x}$. (5 marks)

(b) The radius of a sphere is increased from 10 cm to 10.2 cm. Use differentiation to determine the approximate increase in the surface area, giving the answer as a percentage. (5 marks)

(c) Locate the stationary points of the function $f(x)=x^3-12x^2+36x-6$, and determine its maximum and minimum values. (10 marks)

3. (a) Write down the middle term in the binomial expansion of $(2x+3)^8$ , and determine its value when $x=\dfrac{1}{3}$. (6 marks)

(b) Determine the first four terms in the binomial expansion of $(1-2x)^{\frac{1}{3}}$ (3 marks)

(c) The radius of a cylinder is reduced by 3% and its height is increased by 2%. Use the binomial theorem to determine the approximate percentage change in its:

(i) Volume;
(ii) Curved surface area. (11 marks)

4. (a) Give the matrices $A=\begin{bmatrix}1&-1&0\\1&0&-2\\0&1&1\end{bmatrix}$, and $B=\begin{bmatrix}2&2&1\\1&2&2\\1&3&1\end{bmatrix}$, determine $(AB)^{-1}$. (9 marks)

(b) Three forces $F_1$, $F_2$ and $F_3$ in Newtons, acting an electromechanical system satisfy the simultaneous equations:

$$F_1-2F_2+2F_3=1$$
$$-2F_1+F_2+F_3=2$$
$$-F_1+F_2-3F_3=-5.$$

Use Cramer's rule to solve the equations. (11 marks)

$1-4+4=$

$4+16+25$

$3F_2=14F_3$

5. (a) Determine $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}$ and $\frac{\partial^2 z}{\partial x \partial y}$, given $z = x^2 \cos y$. (5 marks)

(b) (i) Use partial differentiation to determine the value of $\frac{dy}{dx}$ at the point (0,4) for the function $z = x^2 - y^2 + 4xy - 2x + 6y - 8$.

(ii) Hence, determine the equation of the tangent to the curve at this point. (6 marks)

(c) Locate the stationary points of the function $f(x,y) = x^2 - 3y^2 + 6xy - 4x + 12y$, and determine their nature. (9 marks)

6. (a) Determine the values of P and Q such that $5 \sinh x - 3 \cosh x = Pe^{x} + Qe^{-x}$. (4 marks)

(b) Use Osborne's rule to derive identities for:

(i) $\tanh^2 x$;
(ii) $\mathrm{Coth}^2 x$

from the corresponding trigonometric identities. (6 marks)

(c) Three forces $F_1$, $F_2$ and $F_3$ in newtons, acting at a point in a system, satisfy the simultaneous equations:

$$\begin{aligned} 2F_1 - F_2 + 3F_3 &= 4 \\ -F_1 + 2F_2 + F_3 &= 5 \\ F_1 + F_2 - 4F_3 &= 1 \end{aligned}$$

$\frac{\Delta_1}{\Delta}$ $\frac{\Delta_2}{\Delta}$ $\frac{\Delta_3}{\Delta}$

Use substitution method to solve the equations. $A \cdot B = AB\cos\theta$ (10 marks)

7. (a) Given the vectors $\underset{\sim}{A} = -\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, $B = 2\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}$, determine the:

(i) angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$;
(ii) area of the parallelogram whose sides are $\underset{\sim}{A}$ and $\underset{\sim}{B}$. (12 marks)

(b) An electric field has a potential function $\phi(x,y,z) = xy^2 + 2xy + z^2$ in a region of space. Determine, at the point (1,–1,–1):

(i) $|\nabla\phi|$; grad $\hat{A} = \frac{A}{|A|}$
(ii) the direction of maximal increases of $\phi$. (8 marks)

$\phi = \frac{d\theta}{dx} i + \frac{d\theta}{dy} j + \frac{d\theta}{dz} k$ — replace

8. (a) Solve the equation:

$2^{2x+1} - 7(2^x) + 6 = 0.$

$2^{2x} \cdot 2^1 - 7(2^x) + 6 = 0$

let $2^x$

(8 marks)

(b) Evaluate the integrals:

(i) $\int_0^{\pi/2} (2\cos x - 3\sin x + 1)\,dx$;

(ii) $\int_0^1 \frac{(x^{-3/2} + x^{-1/2} + 2)}{x^{-3}}\,dx$.

(7 marks)

(c) Use integration to determine the area of the region bounded by the curve $y = 1 - x^2$, the $x$ - axis and the lines $x = -1$ and $x = 1$. (5 marks)

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